KINETICS OF HEAT TRANSFER BETWEEN A SPHERICAL PARTICLE AND A RAREFIED PLASMA.

1. COLD ION APPROXIMATION

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A kinetic description of heat transfer to a spherical particle from a plasma with allowance for charge-transfer processes is given in the cold ion approximation.

One of the characteristics of plasma heating of particles of materials is that charges – negative electrons and positive ions – take part in heat transfer along with neutral molecules of the plasma-producing gas [1-10]. Plasma electrons colliding with a particle recombine on the surface and are absorbed by the particle, and ions are neutralized by electrons in the material and are scattered by the surface in the form of neutral molecules. Because of the considerable difference between the average velocities of thermal motion of plasma electrons and ions $[\bar{v}_e/\bar{v}_i \sim (m_i T_{e\infty}/m_e T_{i\infty})^{1/2} \gg 1]$, a particle is electrified, acquiring an excess negative charge (potential $\varphi_f < 0$), and a local electric field is formed near the particle, retarding incident electrons and accelerating ions so that the charge fluxes carried by them balance each other.

The kinetic description of the interaction of a particle with a plasma consists of a simultaneous solution of the Boltzmann–Vlasov equations for the velocity distribution functions of molecules, electrons, and ions and the Poisson equation for the electrostatic potential of the plasma. Methods of solving such problems have been developed in the specialized fields of probe diagnostics [11, 12] and astronautics [13], in which principal interest has been devoted to the study of charge transfer, however, whereas the thermal aspects of the interaction of bodies with a plasma, which come to the fore in processes of plasma treatment of materials, have remained little studied. The solution of the kinetic problem is associated with considerable mathematical difficulties and can be carried out in general form only by numerical methods. Analytical results in the problem of heat transfer from a plasma to a particle have been obtained only in certain limiting regimes: for weak ($R \lesssim r_D$) [3, 4, 7-12] and strong ($R \gg r_D$) [5-12] Debye screening in a stationary [3-5, 8] and a moving [6, 7, 9, 10], rarefied, collisionless ($R \ll \ell_j$) plasma.

A rarefied plasma used to process particles of a material may be nonequilibrium one [11, 12]. The simplest form of a nonequilibrium plasma is a two-temperature plasma [11-14], in which the electron temperature considerably exceeds the temperature of the heavy components – molecules and ions ($T_{e\infty} \gg T_{h\infty} \equiv T_{a\infty} = T_{i\infty}$).

In the present paper we describe heat transfer to a spherical particle at rest in a rarefied collisionless plasma, with an arbitrary relationship between the particle radius R and the Debye radius r_D, in the limiting case $T_{i\infty}/T_{e\infty} \to 0$. Solutions obtained for such a temperature ratio of the charge carriers are called the cold ion approximation [14]. Since the time of charge buildup on the particle turns out to be extremely short compared with the characteristic times of thermal and hydrodynamic processes [3-5], heat transfer is analyzed in a regime that is quasi-stationary with respect to the particle potential. Thermoionic emission processes, important only for very high material temperatures and extremely low densities of charge carriers in the plasma [8], are ignored.

The basic equations describing the transfer of charges and energy to a particle in a plasma are written in the following dimensionless form. The Poisson equation, the boundary conditions for the potential in the plasma, and the condition of equality of the electron and ion charge fluxes have the form

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$$\frac{d^2y}{dx^2} = \frac{1}{x_D^2 x^4}(n_i - n_e), \tag{1}$$

$$y(x=0) = 0, \; y(x=1) = y_f. \tag{2}$$

$$j_e^- = (\mu/\tau)^{1/2} j_i^-. \tag{3}$$

Here $x = R/r$, $y = -e\Phi/kT_{e\infty}$, $y_f = -e\varphi_f/kT_{e\infty}$, $n_j = N_j/N_{j\infty}$, $j_j^- = J_j^-/J_j^0$, $x_D = r_D/R$, $\mu = m_e/m_i$, $\tau = T_{e\infty}/T_{i\infty}$, where $J_j^0 = N_{j\infty}(kT_{j\infty}/2\pi m_j)^{1/2}$ and $r_D = (kT_{e\infty}/4\pi e^2 N_{e\infty})^{1/2}$.

The macroscopic parameters of the plasma molecules, electrons, and ions (densities, fluxes, etc.) are determined as moments of different orders of their velocity distribution functions. From the Boltzmann–Vlasov equation it follows that in the free-molecule regime the distribution functions of plasma particles remain constant along their trajectories [11, 12]. Electron and ion trajectories in the field of a charged particle are curved, and in calculating macroscopic parameters it is more convenient to integrate not with respect to velocity space, but in the energy–angular momentum phase plane [15].

Plasma electrons populate two domains in the β–χ phase plane: A_e, corresponding to their trajectories that intersect the particle surface, and B_e, corresponding to trajectories bypassing the particle [here $\beta = \mathscr{E}/kT_{e\infty}$, $\chi = \Omega^2/m_e R^2 kT_{e\infty}$, $\mathscr{E} = (1/2)m_e(v_r^2 + v_t^2) - e\varphi(r)$ and $\Omega = m_e r v_t$]. Domains A_e and B_e correspond to all possible values of $\chi \geq 0$; domain B_e on the β energy scale is located below domain A_e, separated from it by the boundary $\chi = 2(\beta - y_f)$, and is bounded below by the line $\chi = 2(\beta - y)/x^2$. For electrons with a Maxwellian velocity distribution in the unperturbed region of plasma far from the particle, the dimensionless densities $n_j(x)$, charge flux j_j^-, and kinetic flux $e_j^- = E_j^-/E_j^0$ [$E_j^0 = N_{j\infty}kT_{j\infty}(2kT_{j\infty}/\pi m_j)^{1/2}$] are determined by the equations

$$n_e = \frac{\pi x^2}{(2\pi)^{3/2}} \iint\limits_{A_e+B_e} \frac{\nu_K \exp(-\beta)\, d\beta d\chi}{[2(\beta - y) - x^2\chi]^{1/2}}, \tag{4}$$

$$j_e^- = \frac{1}{2} \iint\limits_{A_e} \exp(-\beta)\, d\beta d\chi, \tag{5}$$

$$e_e^- = \frac{1}{4} \iint\limits_{A_e} (\beta - y_f) \exp(-\beta)\, d\beta d\chi. \tag{6}$$

The parameter ν_K (the index K denotes the domain in phase space) from (4) takes the values $\nu_{A_e} = 1$ (only electrons moving toward the particle exist in domain A_e, since emission from its surface is absent) and $\nu_{B_e} = 2$ (both electrons incident on the particle and those reflected by its field exist in domain B_e).

Integration of (4)-(6) results in the equations

$$n_e = \exp(-y) - \frac{1}{2} j_e^- \exp(y_f - y)\, \mathrm{erfc}\,(y_f - y)^{1/2} + $$

$$+ \frac{1}{2} j_e^- (1 - x^2)^{1/2} \exp\left(\frac{y_f - y}{1 - x^2}\right) \mathrm{erfc}\left(\frac{y_f - y}{1 - x^2}\right)^{1/2}, \tag{7}$$

$$j_e^- = \exp(-y_f), \tag{8}$$

$$e_e^- = \exp(-y_f). \tag{9}$$

If the particle's potential is sufficiently high ($y_f \gg 1$), Eq. (7) for the electron density changes into the Boltzmann distribution $n_e = \exp(-y)$.

For ions with a Maxwellian distribution, the equations similar to (7)-(9) for the macroscopic parameters become very cumbersome and contain integrals of the potential with respect to the spatial coordinate, which considerably complicates their use. Various approximations, based on replacing the unperturbed Maxwellian ion distribution by simpler model distributions, have therefore become popular. One such approximation — the cold ion approximation — is analyzed below as applied to the problem of heat transfer between a particle and a plasma.

In the cold ion approximation ($\tau = T_{e\infty}/T_{i\infty} \to \infty$), the ion velocity increases only due to energy acquired in moving in the potential field of the charged particle, so that, using the condition of continuity of the ion flux, we can immediately write

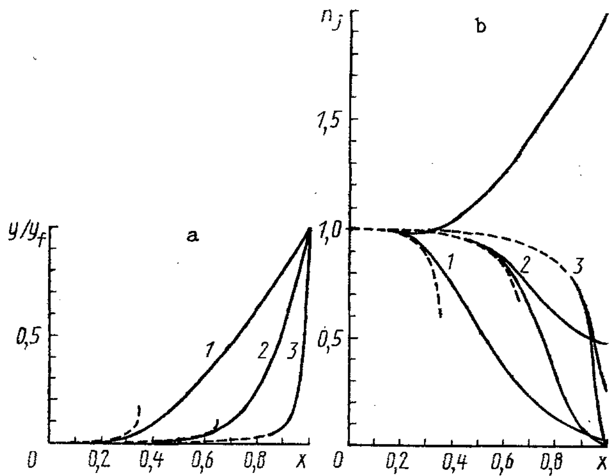

Fig. 1. Spatial distributions of potential y/y_f (a) and density n_j of charge carriers (b) in the vicinity of the particle in a two-temperature ($\tau \to \infty$) argon plasma; the upper branches are curves of density n_i and the lower branches are of density n_e; dashed lines: quasineutral solutions: 1) x_D = 1, y_f = 3.08; 2) 0.1 and 4.34; 3) 0.01 and 4.98.

$$n_i = \frac{1}{2}\frac{j_i^- x^2}{(\pi\tau y)^{1/2}}, \tag{10}$$

$$e_i^- = \frac{1}{2} j_i^- \tau y_f. \tag{11}$$

Equation (10) relates the ion density n_i to the ion flux j_i^- and the plasma potential y at a given point x of the plasma. The ion flux j_i^- as a function of the particle's potential y_f is given by Eqs. (3) and (8). It should also be noted that Eqs. (10) and (11) represent the exact limit as $\tau \to \infty$ of kinetic theory for ions both with monoenergetic [11, 12] and Maxwellian [11, 12, 15] distributions.

The dimensionless total heat fluxes $q_j = Q_j/E_j^0$ of each plasma component are calculated as

$$q_a = 1 - \tau_s, \tag{12}$$

$$q_e = e_e^- + \frac{1}{2} j_e^- w_e, \tag{13}$$

$$q_i = e_i^- + j_i^- \left(\frac{1}{2} w_i - \tau_s\right), \tag{14}$$

where $w_j = W_j/kT_{j\infty}$, $\tau_s = T_s/T_{h\infty}$, $W_e = \Phi_e$, and $W_i = I_i - \Phi_e$. In these equations we allow for the fact that in collisions of electrons and ions with the surface, in addition to the kinetic energy imparted to the particle, the energy of their charge states, corresponding to the electron work function Φ_e and the effective ionization energy at the surface, $I_i - \Phi_e$, is imparted, and heat removal is accomplished by molecules and neutralized ions scattered diffusively by the surface.

The Poisson equation (1), the right side of which depends on the boundary value y(1) = y_f, was solved numerically by the finite-difference method. The equilibrium floating potential y_f of the particle, and hence the fluxes j_j appearing in the equations for the densities n_j, are not known in advance and must be determined in the course of solving the problem. The potential distribution y = y(x) in the plasma was found by choosing values of y_f and dy/dx at the particle surface (x = 1) for which the solution of the Cauchy problem obtained by assigning those values for Eq. (1) satisfies the boundary condition y = 0 at infinity (x = 0).

The spatial distributions of potential and of electron and ion density in a two-temperature argon plasma for different relationships between the Debye radius and the particle radius are shown in Fig. 1. At a sufficient distance from the particle, outside the space-charge layer, these distributions change into the corresponding quasi-neutral solutions of

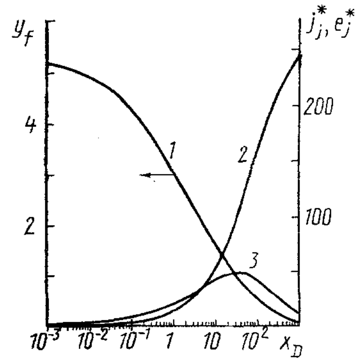

Fig. 2

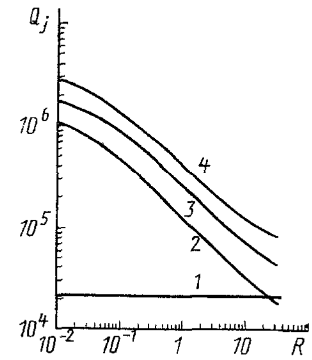

Fig. 3

Fig. 2. Dimensionless floating potential of the particle and dimensionless fluxes of electron and ion charge and energy for a two-temperature ($\tau \to \infty$) argon plasma as a function of the Debye screening parameter: 1) y_f; 2) $j_i^* = j_e^* = e_e^*$; 3) e_i^*.

Fig. 3. Heat transfer to a metallic particle in a low-pressure argon plasma (p = 66 Pa, $T_{e\infty}$ = 15,000 K, $T_{i\infty}$ = 800 K): 1) Q_a; 2) Q_e; 3) Q_i; 4) $Q_t = Q_a + Q_e + Q_i$. Q_j, W/m²; R, μm.

the equation $n_e(x, y) = n_i(x, y)$. In the case of weak Debye screening ($x_D \gg 1$), the ions are efficiently trapped by the particle's field, which penetrates fairly far into the plasma, and their density near the surface exceeds their density in the unperturbed plasma. For strong screening of the electric field ($x_D \ll 1$), the ion density decreases as the particle's surface is approached. The density of electrons, which move in the particle's repulsive field, is lower near the absorbing surface than the unperturbed density in all cases.

In Fig. 2 we give the dimensionless floating potential $y_f = -e\varphi_f/kT_{e\infty}$ of the particle and the dimensionless fluxes of charge $j_j^* = J_j^-/J^*$ and kinetic energy $e_j^* = E_j^-/E^*$ of electrons and ions as a function of the screening parameter $x_D = r_D/R$ of a two-temperature argon plasma, where $J^* = N_{e\infty}(kT_{e\infty}/2\pi m_i)^{1/2}$ and $E^* = N_{e\infty}kT_{e\infty}(2kT_{e\infty}/\pi m_i)^{1/2}$. These quantities are normalized to the respective fluxes calculated from the electron temperature and ion mass, which enables us to reduce the quantities to the same scale, with $j_e^- = \mu^{1/2}j_e^*$, $e_e^- = \mu^{1/2}e_e^*$, $j_i^- = \tau^{1/2}j_i^*$, and $e_i^- = \tau^{3/2}e_i^*$. As x_D decreases from ∞ to 0, the particle's potential y_f undergoes a slow logarithmic increase from 0 to $-(1/2)\ln(2\pi a^2\mu) \simeq 5.18$ [$a = \exp(-1/2) \simeq 0.61$], the fluxes $j_i^* = j_e^* = e_e^*$ decrease from $1/\mu^{1/2} \simeq 271$ to $(2\pi)^{1/2}a \simeq 1.52$, and the flux of ion kinetic energy e_i^* first increases from 0 to a maximum of $\simeq$50 at $x_D \simeq 20$ and then decreases to $(\pi/2)^{1/2}ay_f \simeq 3.94$. This maximum is related to the fact that an increase in the absolute value of the particle's potential leads to an increase in the kinetic energy of the incident ions, on the one hand, and decreases the frequency of their collisions with the surface, on the other, since the fluxes of ion and electron charge must balance each other.

As an illustration of the influence of charge transfer and of the size of the metallic particle on the heat-transfer intensity, in Fig. 3 we give calculated results for a low-pressure argon plasma [16] with the parameters p = 66 Pa, $N_{e\infty} = 6.5 \cdot 10^{18}$ m^{-3}, $T_{e\infty}$ = 15,000 K, and $T_{i\infty}$ = 800 K. In such a plasma the degree of ionization is $\eta \simeq 0.001$, the mean free path is $\ell_j > 300$ μm, and the Debye radius is $r_D \simeq 3$ μm. Despite the low degree of ionization, the contribution of electrons and ions to the heat flux from the plasma to the particle considerably exceeds the contribution of neutral molecules. Charge-transfer processes are especially important in heat transfer for small particles, for which Debye screening of the electric field by the plasma turns out to be weak. The simplest estimates, based on the Richardson formula, show that under these conditions electron thermionic emission begins to affect the results at particle temperature T_s exceeding 2000-3000 K, depending on the particle's size and floating potential.

The factors that determine the considerable contribution of electrons and ions to heat transfer to a particle in a two-temperature plasma include the following: 1) electrization of the particle and penetration of its electric field into the plasma, leading to an increase in the ion flux to the surface; 2) a considerable increase in the kinetic energy acquired by the ions in the potential field of the charged particle over the average energy of their thermal motion; 3) the energies of the charged states of electrons and ions and the energy of electron thermal motion considerably exceed the thermal energy of the molecules.

NOTATION

e, electron charge; $\mathscr{E}$, total energy; E_j^-, kinetic energy flux density; I_i, ionization energy; J_j^-, number flux density of plasma particles; k, Boltzmann constant; ℓ_j, mean free path; m_j, mass; N_j, calculated density; p, pressure; Q_j, heat-flux density; r, spatial coordinate; r_D, Debye radius; R, particle's radius; T_j, temperature; v, velocity; η, degree of ionization; φ, plasma potential; φ_f, floating potential of the particle; Φ_e, electron work function; Ω, angular momentum. Indices: a, molecules; e, electrons; i, ions; h, heavy plasma particles (molecules and ions); r, radial component; s, surface; ∞, unperturbed region of plasma far from the particle; t, tangential component; -, direction toward the particle.

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